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which is the equation of a straight line through A and B. To find the point in which this line cuts the axis of y, set x equal to zero and solve the resulting equation for y. Since $r^2 - \alpha^2$ is independent of the position of the variable circle, it follows that the intersection of the chord AB and the perpendicular to l through P is fixed.

It is interesting to note that the above proof still holds when the intersections of the two circles are imaginary points.

Also solved by H. C. Feemster, Oscar S. Adams, Harold R. Schaufler, A. M. Harding, Paul Capron, Margaret F. Willcox, and Roger A. Johnson.

522 (Geometry). Proposed by GEORGE Y. SOSNOW, Newark, N. J.

Prove that the sum of the squares of the edges of a tetrahedron is equal to four times the sum of the squares of the lines joining the middle points of the opposite edges.

Solution by R. M. Mathews, Riverside, California.

Let $ABCD$ be the tetrahedron with X, the mid-point of BD, opposite to Y, the mid-point of AC. In the triangle BYD, we have

$$2XY^2 = BY^2 + DY^2 - 2BX^2,$$

from the theorem: The sum of the squares on two sides of a triangle is equal to twice the square on half the third side plus twice the square on the median to that side.

By the same theorem, we find $2BY^2$ in the triangle ABC and $2DY^2$ in the triangle ADC. Then,

$$4XY^2 = AB^2 + BC^2 + CD^2 + DA^2 - AC^2 - BD^2.$$

Similarly with U, the middle point of AB, and V, of DC:

$$4UV^2 = AC^2 + CB^2 + BD^2 + DA^2 - AB^2 - DC^2;$$

and with T, the middle point of AD, and W, of BC:

$$4TW^2 = AC^2 + CD^2 + DB^2 + BA^2 - AD^2 - BC^2.$$

By addition, we have

$$4(XY^2 + UV^2 + TW^2) = AB^2 + BC^2 + CD^2 + AD^2 + AC^2 + BD^2.$$

Remark: Considering any one of the three equations above we have proved the theorem: Four times the square on the median joining two opposite edges of a tetrahedron is equal to the sum of the squares on the other edges minus the sum of the squares on the edges which it joins.

Also solved by Horace Olson, Oscar S. Adams, J. L. Riley, and H. C. Gossard.

345 (Mechanics). Proposed by J. L. RILEY, Northeastern State Normal School, Tahlequah, Okla.

Two particles A and B are together in a smooth circular tube. A attracts B with a force whose acceleration is ω^2 and moves along the tube with uniform angular velocity 2ω, B being initially at rest; prove that the angle φ subtended by AB at the center after a time t is given by the equation

$$\log \tan \frac{\pi + \varphi}{4} = \omega t.$$

Solution by William Hoover, Columbus, Ohio.

Let B' be the initial positions of A and B, and put $BB' = s$, and $AB = x$.

Resolving forces tangentially,

$$\frac{d^2s}{dt^2} = \omega^2 x \cos ABC, \tag{1}$$

BC being the tangent through B.

Now if O be the center of the circle,

$$\angle AOB = \frac{2r\omega t - s}{r}, \qquad \angle ABC = \frac{2r\omega t - s}{2r},$$

$$x \cos ABC = 2r \sin \tfrac{1}{2}AOB \cos ABC = 2r \sin \frac{2r\omega t - s}{2r} \cos \frac{2r\omega t - s}{2r} = r \sin \frac{2r\omega t - s}{r}$$

$$= r \sin (2\omega t - \theta),\ \theta \text{ being the angle } BOB' \text{ and } s = r\theta.$$

Substituting these in (1),

$$\frac{d^2(2\omega t - \theta)}{dt^2} = -\,\omega^2 \sin(2\omega t - \theta), \tag{2}$$

or, since $2\omega t - \theta = \varphi$,

$$\frac{d^2\varphi}{dt^2} = -\,\omega^2 \sin \varphi. \tag{3}$$

Multiplying by $2(d\varphi/dt)$ and integrating,

$$\frac{d\varphi^2}{dt^2} = 2\omega^2 \cos\varphi + C. \tag{4}$$

When $\varphi = 0$, $d\varphi/dt = 0$; hence, $C = 2\omega^2$, and (4) is

$$\frac{d\varphi^2}{dt^2} = 2\omega^2(1 + \cos\varphi) = 4\omega^2 \cos^2 \tfrac{1}{2}\varphi. \tag{5}$$

This gives

$$\omega dt = \frac{d\frac{1}{2}\varphi}{\cos\frac{1}{2}\varphi}; \tag{6}$$

integrating and using the fact that $\varphi = 0$ when $t = 0$, we have

$$\log\tan\frac{\pi + \varphi}{4} = \omega t. \tag{7}$$

Also solved by G. Paaswell and J. B. Reynolds.

346 (Mechanics). Proposed by WILLIAM HOOVER, Columbus, Ohio.

Half the length of one of the equal parts of a uniform heavy string resting in equilibrium over a smooth horizontal indefinitely thin peg is cut off; determine the instantaneous change in the pressure on the peg.

Solution by the Proposer.

Let m = the mass of a unit of length of the string, $2a$ = the whole length of the string, and so a, $a/2$, the parts at the instant of cutting, $3a/2$ the length in motion after any time t from the beginning of motion, x = the longer part at the same instant, and T = the corresponding tension in the string; then the equations of motion are, noticing that the momenta of the moving masses are each variable,

$$\frac{d}{dt}\left(mx\frac{dx}{dt}\right) = m\left(\frac{dx^2}{dt^2} + x\frac{d^2x}{dt^2}\right) = mgx - T, \tag{1}$$

$$\frac{d}{dt}\left\{ m(\tfrac{3}{2}a - x)\frac{d}{dt}(\tfrac{3}{2}a - x) \right\} = \frac{d}{dt}\left\{ -\,m\tfrac{3}{2}a\frac{dx}{dt} + mx\frac{dx}{dt} \right\} = mg(\tfrac{3}{2}a - x) - T. \tag{2}$$

Subtracting (2) from (1), we have

$$\frac{d^2x}{dt^2} = \frac{2g}{3a}\left(2x - \frac{3a}{2}\right). \tag{3}$$

Multiplying by $2(dx/dt)$ and integrating, noticing that when $x = a$, $(dx/dt) = 0$, we obtain

$$\frac{dx^2}{dt^2} = \frac{g}{3a}\left\{\left(2x - \frac{3a}{2}\right)^2 - \frac{a^2}{4}\right\}. \tag{4}$$

Substituting the values of dx^2/dt^2 and d^2x/dt^2 in (1), and putting $T = T_0$ when $x = a$, we find $T_0 = \tfrac{2}{3}mga$.

If P = the required initial pressure, $P = 2T_0 = \tfrac{4}{3}mga = 2mga - \tfrac{1}{3}\cdot 2mga$, so that the pressure before cutting is diminished by one third.

Also solved by Horace Olson.

264 (Number Theory). Proposed by C. F. GUMMER, Kingston, Canada.

Find a general formula for three squares in arithmetical progression. Is it possible for the common difference to be a perfect square?